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CONTACT PROBLEMS FOR SYSTEMS OF ELASTIC HALF-PLANES*

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Static and stationary dynamic problems are considered for systems of N elastic isotropic half-planes attached by arbitrary sections of their boundaries. Outside the attached sections the half-planes are contiguous to stamps and flexible facings. The kinds of mixed boundary conditions, whose number can reach six, are given in each half-plane independently. In particular, the most important case of a plane $N = 2$ does not require the presence of specular symmetry of the types on opposite edges of slits, which provides the possibility of studying new classes of problems of the cutting, wedging, and debonding of inclusions.

The procedure proposed for the solution enables the problems in question to be reduced, in a general formulation, to Hilbert-Riemann bounary-value problems on N-sheeted Riemann surfaces defined by bifurcation and the law of attachement of the sheets. If the problem of constructing the algebraic function of the Riemann surface obtained along its bifurcation is solved (for $N = 2$, i.e., for a hyperelliptic surface, this function is well-known for $N \leqslant 4$ and it can obviously also be found in the general case), then the corresponding contact problem is solved by quadratures. Examples are considered.

1. Let $\{R_k\}_1^N$ be a set of specimens of the complex plane $z = x + iy$; $S_k = \{z \in R_k: y > 0\}$, $k = 1, 2, \ldots, N^+$, $N^+ < N$, is the upper elastic half-plane $S_k = \{z \in R_k: y < 0\}$, $k = N^+ + 1, N^+ + 2, \ldots, N$ is the lower elastic half-plane, and Γ_k is the boundary of S_k. Each k-th upper half-plane is contiguous to $N_k \in [1, N^-]$, $N^- = N - N^+$, by some lower half-planes, $\Gamma_{kl}' \subset \Gamma_k$ and $\Gamma_{lk}' \subset \Gamma_l$ are contact boundaries of the domains S_k and S_l and coincide when R_k and R_l are superimposed $\Gamma_{kl}' \cap \Gamma_{km}' = \varnothing$ for $l \neq m$. Let the elastic domain $S_1 \cup S_2 \cup \ldots S_N$ "be connected" in the sense that for any k and $l \in [1, N]$ a chain of boundaries $\Gamma_{kr}', \Gamma_{rs}', \ldots, \Gamma_{qp}', \Gamma_{pl}'$ connecting the domains S_k and S_l exists: Γ_k' is the union of Γ_{kl}' over all N_k values of l, $\Gamma_k'' = \Gamma_k \setminus \Gamma_k'$, μ is the shear modulus, ν is Poisson's ratio, and ρ is the density of the material S. In the general case different kinds of fundamental or mixed boundary conditions P_k for different k are formulated on Γ_k and the nature of the singularities allowable at the separation points is indicated; stress field intensities satisfying the equilibrium and connectedness conditions of the domain S are given at infinity in each half-plane. The boundaries Γ_k move with the identical constant subsonic velocity $c \geqslant 0$ relative to the fixed domains S_k. It is required to determine the elastic deformations of the domain S.

In the case $N = 2$ certain fundamental problems for a homogeneous plane with slits are solved by quadratures for $P_1 = P_2$ and $P_1 \neq P_2$ /1/, for a composite plane and for $P_1 = P_2$ /2/; mixed problems are solved in the same form just for $P_1 = P_2$ /3-6/, i.e., for specularly symmetric kinds of conditions on opposite edges of the slits.

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The independent kinds of conditions $P_k \neq P_l$, $k \neq l$, $N \geqslant 2$, under consideration in this paper occur in a study of the cutting, wedging, and debonding of inclusions and facings, and slipping and rolling of several elastic discs on a half-space. The purpose of this paper is to describe methods that enable the problem to be reduced to Hilbert-Riemann boundary-value problems on an N-sheeted Riemann surface and solving it by quadratures.

Let us partition the plane P_k along the lines Γ_k', $k = 1, 2, \ldots, N$ and let us glue the slit edges Γ_k' crosswise in R_k and Γ_{lk}' in R_l for all $k \leqslant N^+$ and N_k values of l for each k. We carry the notation for the lines, half-planes and planes from Sect.1 over to the N-sheeted Riemann surface formed in this manner and governed by a certain irreducible polynomial $F(z, w)$ and the fundamental variable z. We will consider the elastic domain S as part of the Riemann surface R with the edges $\Gamma'' = \Gamma_1'' \cup \Gamma_2'' \cup \ldots \Gamma_N''$. The subsequent constructions are carried out on R and S.

2. Let the solution of the elasticity theory problem in the half-plane S_k, representable in terms of one function $\Phi_k(z)$, piecewise-analytic in the plane R_k, have the following form on Γ_k:

$$(u' + iv')_k^{\pm}(x) = \Psi_1[x, \Phi_k^{\pm}(x), \Phi_k^{\pm}(x)], \; f' \equiv \partial f/\partial x \qquad (2.1)$$
$$(\sigma - i\tau)_k^{\pm}(x) = \Psi_2[x, \Phi_k^{\pm}(x), \Phi_k^{\mp}(x)]$$

and generate Hilbert and Riemann boundary conditions in R_k

$$\operatorname{Im}[p_k^{\pm}(x)\Phi_k^{\pm}(x)] = g_k^{\pm}(x), \quad x \in E_k^1; \quad \Phi_k^+(x) = p_k(x)\Phi^-(x) + g_k(x), \quad x \in E_k^2 \qquad (2.2)$$

under conditions P_k on Γ_k'' independently of the conditions on Γ_k'.

Here u_k, v_k, $\tau \equiv \tau_{xy}$, $\sigma \equiv \sigma_y$ are the tangential and normal displacements and stresses; the given coefficients and free terms satisfy the usual constraints /7/ $\Gamma_k'' = E_k^{-1} \cup E_k^2$, $\Phi_k^{\pm}(x)$ are limit values of the function $\Phi_k(z)$ on Γ_k'', $k = 1, 2, \ldots, N$.

Then under total adhesion conditions on Γ_{kl}' $(k \leqslant N^+)$

$$(u' + iv')_k^+(x) = (u' + iv')_l^-(x), \; (\sigma - i\tau)_k^+(x) = (\sigma - i\tau)_l^-(x) \qquad (2.3)$$

the problem of Sect.1 on S reduces to an analogue of problem (2.2) on R.

Indeed, we introduce the function $\Phi(z, w) = \Phi_k(z)$, $(z, w) \in R_k$, $k = 1, 2, \ldots N$ analytic in $R \setminus \Gamma''$, $\Gamma'' = \Gamma_1'' \cup \Gamma_2'' \cup \ldots \Gamma_N''$. For crosswise glueing $\Phi_k^{\pm}(x) = \Phi_l^{\mp}(x)$ on Γ_{kl}' and Γ_{lk}' by virtue of the analyticity of $\Phi(z, w)$ on $\Gamma' = \Gamma_1' \cup \Gamma_2' \cup \ldots \Gamma_N'$. Consequently, on substituting (2.1) into conditions (2.3), the latter are satisfied identically, and conditions (2.2) become equivalent to the following conditions on the Riemann surface R with slits E^1:

$$\operatorname{Im}[p^{\pm}(x, v)\Phi^{\pm}(x, v)] = g^{\pm}(x, v), \; (x, v) \in E^1 \qquad (2.4)$$
$$\Phi^+(x, v) = p(x, v)\Phi^-(x, v) + g(x, v), \; (x, v) \in E^2$$
$$p^{\pm}(x, v) = p_k^{\pm}(x), \; g^{\pm}(x, v) = g_k^{\pm}(x), \; (x, v) \in E_k^1$$
$$p(x, v) = p_k(x), \; g(x, v) = g_k(x), \; (x, v) \in E_k^2$$
$$E^r = E_1^r \cup E_2^r \cup \ldots E_N^r, \; r = 1, 2, \ldots$$

Here $\Phi^{\pm}(x, v)$ are limit values of the function $\Phi(z, w)$ on $\Gamma'' = E^1 \cup E^2$ a multiple divisor consistent with the nature of the elastic stresses at singular points of the domain S.

If the function $F(z, w)$ is constructed and $E^1 = \varnothing$, then the Riemann problem (2.4) is solved in quadratures on R; if $E^1 \neq \varnothing$, then the Hilbert-Riemann problem can again be reduced to a Riemann problem on a $2N$-sheeted closed Riemann surface constructed in the form of a double of the surface R with edge E^1 /3/.

3. Let us consider the static deformations by setting $c = \rho = 0$. The Muskhelishvili solution /1/

$$(u' + iv')_k(z) = \Phi_k(z) - \Phi_k(\bar{z}) + (z - \bar{z})\overline{\Phi_k'(z)}, \; z \in S_k \qquad (3.1)$$
$$2\mu(\sigma - i\tau)_k(z) = \varkappa\Phi_k(z) + \Phi_k(\bar{z}) - (z - \bar{z})\Phi'(z), \; \varkappa = 3-4\nu$$

has the form (2.1) and under mixed conditions (homogeneous for brevity) of four kinds

$$u_k' = \sigma_k = 0, \; x \in E_k^1; \; u_k' = v_k' = 0, \; x \in E_k^2 \qquad (3.2)$$
$$v_k' = \tau_k = 0, \; x \in E_k^3; \; \tau_k = \sigma_k = 0, \; x \in E_k^4;$$
$$\Gamma_k'' = E_k^1 \cup E_k^2 \cup E_k^3 \cup E_k^4$$

results in the problem (2.2) /8/

$$\operatorname{Re}\Phi_k^{\pm}(x) = 0, \; x \in E_k^1; \; \Phi_k^+(x) + \varkappa\Phi_k^-(x) = 0, \; x \in E_k^2 \qquad (3.3)$$
$$\operatorname{Im}\Phi_k^{\pm}(x) = 0, \; x \in E_k^3; \; \Phi_k^+(x) - \Phi_k^-(x) = 0, \; x \in E_k^4$$

independently of the conditions on Γ_k'.

According to the connections (2.2)-(2.4), we obtain the Hilbert-Riemann problem

$$\begin{aligned} &\operatorname{Re}\Phi^{\pm}(x, v) = 0,\ (x, v) \in E^1;\ \Phi^+(x, v) + \varkappa\Phi^-(x, v) = 0,\ (x, v) \in E^2 \\ &\operatorname{Im}\Phi^{\pm}(x, v) = 0,\ (x, v) \in E^3;\ \Phi^+(x, v) = \Phi^-(x, v),\ (x, v) \in E^4 \end{aligned} \tag{3.4}$$

for the function $\Phi(z\ w)$ on R upon total adhesion of the domains S_k and (3.3).

If the boundary Γ'' of the domain S is stress-free, then according to (3.2)-(3.4) $\Gamma'' = E^4$, $\Phi^+(x, v) = \Phi^-(x, v)$ on Γ'', therefore, the functions $\Phi(z, w)$ is analytic in $R \setminus \Gamma'$, and meromorphic in R; an analogous result can be obtained for other kinds of fundamental conditions (3.2) by transforming the form (3.1). At the same time for $N = 2$ the usual means /1/ results in a Riemann problem in $\Phi_1(z)$. Under mixed conditions the boundary-value problem (3.4) also turns out to be simpler than the alternative problem in the complex plane if it exists because of the construction of the function $F(z, w)$.

We will show this by exmaples. Let S be an elastic plane with the slits $\Gamma_{12}' = [a_1, b_1] \cup [a_2, b_2] \cup \ldots [a_t, b_t]$. Then $N = 2$ and therefore R is a hyperelliptic Riemann surface and

$$F(z, w) = w^2 - \prod_{k=1}^{t}(z - a_k)(z - b_k)$$

Example 1. Let $t = 1, a_1 = -a, b_1 = a > 0$, and the slits $(-\infty, -a), (a, \infty)$ stress-free. In this case, as is noted, problem (3.4) does not occur. In the class E of stresses integrable on continuations of the slits and bounded at infinity, the general solution (3.1) is expressed in terms of themeromorphic function

$$\Phi(z, w) = (Az + B)w^{-1} + C,\ w = \pm\sqrt{z^2 - 1},\ (z, w) \in S_{1,2}$$

We find the complex constants A, B, C by means of two components of the principal vector given at infinity in S_1 and the quantities: the principal moment in S_1, the general rotation ε^∞ in S, and the constant tensile stresses σ_{xk}^∞ in S_k.

Example 2. Let $t = 1, a_1 = -\infty, b_1 = 0$, let the upper edge of the slit $(0, \infty)$ be free, $\Gamma_1'' = E_1^4$, and let the lower edge be completely attached to a rectilinear stamp $E_2^2 = \{[a, b]\}$, $a \in (0, \infty)$; $E_2^1 = E_2^3 = \varnothing$. According to (3.4), for the piecewise-analytic function $\Phi(z, w)$, $w = \pm\sqrt{z}$, $(z, w) \in S_{1,2}$, having the boundary E_2^2 and the pole $z = 0$, we obtain the Riemann problem $\Phi^+(x, v) + \varkappa\Phi^-(x, v) = 0$, $(x, v) \in E_2^2$. The function $z = z_1^2$ maps R conformally into the complex plane z_1, where the Riemann problem takes the form $\Phi^+(x_1) + \varkappa\Phi^-(x_1) = 0$, $x_1 \in [\sqrt{a}, \sqrt{b}]$. Writing down its solution /7/ and returning to R, we obtain the function

$$\Phi(z, w) = w^{-1}(w - \sqrt{a})^{-1/2 + i\beta}(w - \sqrt{b})^{-1/2 - i\beta}(Aw^2 + Bw + C),$$
$$\beta = \ln(\varkappa/(2\pi))$$

that determines the general solution (3.1) in E. The complex constants A, B, C can be found from two components of the forces (X, Y) applied to the stamp in the quantities $\varepsilon^\infty, \sigma_{x1}^\infty = \sigma_{x2}^\infty = \sigma^\infty$ and two stress intensity factors $N_{\rm I}, N_{\rm II}$ that decreases as $z^{-1/2}$ as $z \to \infty$. In particular if $\varepsilon^\infty = \sigma^\infty = N_{\rm I} = N_{\rm II} = 0$, then $A = B = 0$, $C = -(4\pi)^{-1}(X + iY)$ and the contact stresses under the stamp have the form

$$(\sigma - i\tau)_2(x) = \frac{C(\varkappa + 1)(i\cos\psi - \sin\psi)}{\sqrt{\varkappa x(\sqrt{x} - \sqrt{a})(\sqrt{b} - \sqrt{x})}},\quad \psi = \beta\ln\frac{(x - a)\sqrt{\sqrt{x} + \sqrt{b}}}{(b - x)\sqrt{\sqrt{x} + \sqrt{a}}}$$

Example 3. We change only the condition under the stamp to a sliding contact condition in Example 2. According to (3.2)-(3.4), in this case $E_2^3 = \{[a, b]\}$, and the function $\Phi(z, w)$, $w^2 = z$ is a solution of the Hilbert problem on $R \setminus E_2^3$: $\operatorname{Im}\Phi^{\pm}(x, v) = 0$, $(x, v) \in E_2^3$. Using the preceding method, we obtain in E

$$\Phi(z, w) = iw^{-1}(w - \sqrt{a})^{-1/2}(w - \sqrt{b})^{-1/2}(A_1w^2 + A_2w + A_3) +$$
$$A_4w^{-1} + A_5)$$

The real constants $A_1, \ldots, A_5$ can be found from the same conditions as in Example 2 beause now $X = 0$. If $\varepsilon^\infty = \sigma^\infty = N_{\rm I} = N_{\rm II} = 0$ then $A_s = 0$ for $s \neq 3$, $A_3 = -(4\pi)^{-1}Y$, under the stamp $\sigma(x) = Y[2\pi\sqrt{x(\sqrt{x} - \sqrt{a})(\sqrt{b} - \sqrt{x})}]^{-1}$.

Remark. Mixed problems of the type 2 and 3 (with one semi-infinite slit and arbitrary conditions (3.2) on the slit), can also be solved by the conformal mapping $z = z_1^2$ for the domain S into the half-plane z_1.

4. Considering dynamic stationary deformations in the subsonic mode, we set $\rho > 0$, $0 < c < c_2$, $c_2^2 = \mu\rho^{-1}$ in Sect.1. This problem has a solution of the form (2.1) /6/

$$\mu u_k' = -\mathrm{Re}\,[\varphi_{k1}(z_1) + q_2\varphi_{k2}(z_2)],\quad \mu v_k' = \mathrm{Im}\,[q_1\varphi_{k1}(z_1) + \varphi_{k2}(z_2)]$$
$$\sigma_k = 2\mathrm{Re}\,[q\varphi_{k1}(z_1) + q_2\varphi_{k2}(z_2)],\quad \tau_k = 2\,\mathrm{Im}\,[q_1\varphi_{k1}(z_1) + q\varphi_{k2}(z_2)]$$
$$\varphi_{ks}(z) = q_s^{-1/2}[(-1)^{s+1}R^+\Phi(z) + R^-\overline{\Phi(\bar z)}],\quad R^\pm = \sqrt{q_1q_2} \pm q,\ z_s = x + iq_sy$$
$$q_s = \sqrt{1 - c^2c_s^{-2}},\ 2q = 1 + q_2^2,\ c_1^2 = 2(1-\nu)(1-2\nu)^{-1}c_2^2,\ s = 1, 2$$

and under the conditions (3.2) reduces to the Hilbert-Riemann Eqs.(2.2). For total adhesion (3.3) on Γ' the appropriate Hilbert-Riemann problem for the function $\Phi(z, w)$ analytic in $R \setminus (\Gamma'' \setminus E^4)$, can be written in the form

$$\mathrm{Re}\,\Phi^\pm(x, v) = 0,\ (x, v) \in E^1;\ \mathrm{Im}\,\Phi_*^\pm(x, v) = 0,\ (x, v) \in E^3$$
$$\Phi^+(x, v) + Q\Phi^-(x, v) = 0,\ (x, v) \in E^2;\ Q = R^+(1 - \sqrt{q_1q_2})(R^-(1 + \sqrt{q_1q_2}))^{-1}$$

5. Let us consider the solution of the static and dynamic problems of Sect.1 that are expressed in terms of two analytic functions $\Phi_{kr}(z)$, $r = 1, 2$ in the Galin form and take the following form on the domain boundary /9, 10/

$$u_k'(x) = \mathrm{Re}\,[a_1\Phi_{k1}(x) + b\Phi_{k2}(x)],\quad \sigma_k(x) = \mathrm{Re}\,\Phi_{k2}(x) \tag{5.1}$$
$$v_k'(x) = \mathrm{Im}\,[b\Phi_{k1}(x) + a_2\Phi_{k2}(x)],\quad \tau_k(x) = \mathrm{Im}\,\Phi_{k1}(x)$$

where $f' = \partial f/\partial x$, $4\mu a_j = 1 + \varkappa$, $4\mu b = 1 - \varkappa$; $j = 1, 2$; , for $c = 0$ and $f' = \partial f/\partial t$, $2\mu R^+R^-b = q_1q_2 - q$, $2\mu R^+R^-a_j = q_j(1 - q)$ for $c \in (0, c_2)$

These solutions allow the formulation of other important boundary conditions on the contact lines, particularly for the theory of cutting /11/

$$[u'] = [v'] = [\sigma] = [\tau] = 0,\ \ x \in D_{kl}^3;\ \ [v'] = [\sigma] = \tau_k + \rho_{kl}\sigma_k = \tag{5.2}$$
$$[\tau] = 0,\ x \in D_{kl}^2;\ \ [u'] = [\tau] = \sigma_k = \sigma_l = 0,\ \ x \in D_{kl}^1$$
$$\Gamma_{kl}' = D_{kl}' \cup D_{kl}^2 \cup D_{kl}^3;\ \ [f] = f_k - f_l,\ \ x \in \Gamma_{kl}'$$

and outside the contact

$$\sigma_k = \tau_k = 0,\ x \in E_k^1;\ v_k' = \tau_k + \rho_k\sigma_k,\ x \in E_k^2 \tag{5.3}$$
$$u_k' = \sigma_k = 0,\ \ x \in E_k^3;\ \ E_k'' = E_k^1 \cup E_k^2 \cup E_k^3$$

where $\rho_{kl} \equiv \rho_{kl}(x)$, $\rho_k \equiv \rho_k(x)$ are the coefficients of dry friction.

Indeed by introducing analytic functions $\Phi_r(z, w) = \Phi_{kr}(z)$, $(z, w) \in S_k$, on the N-sheeted Riemann surfaces with edges $S^r = S \setminus D^{3-r}$, $D^r = \bigcup_{k,l} D_{kl}^r$, $r = 1, 2$, we obtain two scalar Hilbert problems

$$\mathrm{Re}\,\Phi_2 = 0;\ \ (x, v) \in D^1 \cup E^1 \cup E^3;\ \ a_2\,\mathrm{Im}\,\Phi_2 - b\rho_*\,\mathrm{Re}\,\Phi_2 = 0,$$
$$(x, v) \in E^2$$
$$\mathrm{Im}\,\Phi_1 = -\rho_*\,\mathrm{Re}\,\Phi_2\quad (x, v) \in D^2 \cup E^2;\ \ \mathrm{Im}\,\Phi_1 = 0,\ \ (x, v) \in E^1$$
$$\mathrm{Re}\,\Phi_1 = 0,\ (x, v) \in E^3;\ \ \Phi_k \equiv \Phi_k(x, v);\ \rho_* \equiv \rho_k(x),\ (x, v) \in E_k^2$$
$$\rho_* \equiv \rho_{kl}(x),\ (x, v) \in D_{kl}^2$$

on S^r from conditions (5.1)-(5.3).

These problems can be reduced to Riemann problems on N-sheeted closed surfaces that are doubles of S^r /3/. For $N = 2$ they are Hilbert problems in the complex plane with slits for the functions $\Phi_r(z) = \Phi_{kr}(z)$, $z \in S_k$, $k, r = 1, 2$, and can be solved by a different method /5, 6/.

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THE PLANE CONTACT PROBLEM FOR AN ELASTIC LAYER FOR HIGH VIBRATION FREQUENCIES*

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The problem of stamp vibrations on the surface of an elastic strip located on a stiff base is examined. There is no friction in the contact domain or between the strip and the base. It is noted that the use of methods known earlier at high frequencies results in the need to solve linear algebraic systems of very high order. A method which enables the shortwave asymptotic form of the solution to be written in an explicit form convenient for qualitative and quantitative analyses is proposed.

1. We will assume that the time-dependence of all the functions occurring in the solution of the problem has the form $f(x, t) = \operatorname{Re}[f(x) e^{-i\omega t}]$ (ω is the angular frequency of the vibrations). Then the problem under investigation can be reduced to an integral equation in the unknown contact stress $p(x)$ referred to $\mu W/h$ /1/

$$\int_{-a}^{a} p(\xi) K(x-\xi) d\xi = 1, \quad |x| < a \tag{1.1}$$

$$K(x) = \frac{1}{2\pi} \int_{\Gamma} L(u) e^{-i\varkappa x u} du, \quad L(u) = L_1(u) - L_2(u)$$

$$L_1(u) = \sigma_1/\Delta(u), \ L_2(u) = \sigma_1 P_1(u)/\Delta(u)$$

$$P_1(u) = e^{-2\varkappa\sigma_1} + e^{-2\varkappa\sigma_2} - e^{-2\varkappa(\sigma_1+\sigma_2)}$$

$$\Delta(u) = 4u^2\sigma_1\sigma_2 G_1(u) F_2(u) - (2u^2-1)^2 G_2(u) F_1(u)$$

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